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MATHEMATICAL PROGRAMMING AND THE AIRCRAFT SQUADRON

MARVIN M. QUAID

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MATHEMATICAL PROGRAMMING AND THE AIRCRAFT SQUADRON

By

Marvin M. Quaid
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The tensions of a decade of cold war coupled with the increase in the technology of aerial warfare and its attendant complexities have made the task of managing an aircraft squadron much more difficult. The most pressing problem facing an aviation unit commander is the optimum use of resources in order to maximize the unit's efficiency and effectiveness.

This paper proposes that mathematical programs can be utilized in the solution of problems of this nature. Some illustrations are made in which elementary techniques of the mathematician, operations analyst, and industrial manager are applied to an aircraft squadron. Some of the problems of a fictitious squadron during its training phase are used as a vehicle for illustrating the use of mathematics in obtaining the optimum use of resources. In the example used, time is found to be the constraining factor.

The purpose is to show the applicability of mathematical techniques to an aircraft squadron and to encourage the use of the discipline of thought inherent in such techniques.

May 1962
Master of Science in Management
Navy Management School

MATHEMATICAL PROGRAMMING

AND

THE AIRCRAFT SQUADRON

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A Research Paper

Presented to

the Faculty of the Navy Management School

U. S. Naval Postgraduate School

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In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Management

* * * * *

by

Marvin M. Quaid, LCDR, USN

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CHAPTER I

INTRODUCTION AND PROBLEM DEFINITION

It is often said that every naval officer is a manager first and a technician second. While there can be little quarrel with this concept, in actuality it is often assumed that naval officers will be good managers by virtue of the fact they must manage. Often these officers are poorly equipped either by lack of experience, background or education to cope with the problems they must solve. This is by no means an indictment of the assignment of officers to tasks within the Naval Establishment, but rather a relatively trite observation. Some might argue that a naval officer's education should be more heavily slanted toward managerial skills in his undergraduate curriculum. Again, it might be argued with equal merit that our younger officers must have an increasingly specific technical knowledge in order to be able to operate the equipment entrusted to their care. In this paper I shall not indulge in such arguments however valid they may be, but rather attempt to aid fleet managers in solving their problems.

I shall further limit my discussion to aircraft squadrons and the problems that typically beset them. Aircraft squadrons are a particularly fertile field for applying some of the more recently developed techniques, because they are by their very

nature subject to a considerable number of constraints. The satisfaction of requirements within the constraints established either by higher authority, by the unit, or by the nature of the problem is indeed a delicate task and demands exceptional managerial ability. Aircraft squadrons as well as the entire Navy are being subjected to increasing pressure for greater efficiency. The past decade of cold war has increased the size and complexity of our Naval Air Forces and has added to these pressures. Shortage of personnel is an ever-present constraint on the efficiency of a squadron and shortage of operating funds has been a recurring problem. Perhaps the greatest pressure has been exerted by the increased tempo of operations. There is an ever-diminishing amount of slack time available to squadron personnel. Training and maintenance are severely affected by this shortage of time.

Another reason for this attempt to aid the fleet aircraft squadron is that such a unit is normally staffed in large measure by relatively junior, inexperienced officers. While these officers have had extremely thorough technical training, most have had virtually no management training of any type. Thus by instilling good practices into these units a double benefit will be reaped; the units will become more effective, and the junior officers will get some management training early in their careers. The

problem, then, is to attain the highest state of training and maintenance possible under the constraints. The method proposed by this paper to accomplish this optimum is to apply some of the recently developed techniques. Special emphasis is to be placed on the rationale, or discipline of thought, of the operations analyst and industrial manager.

Unfortunately the reader will not find in these pages a set of formulae that will optimize a squadron's effectiveness. Even the most sophisticated mathematical model is incapable of evaluating all the subtleties and personal factors that the average officer in a squadron must weigh in his decision making. However, tremendous strides are being made in mathematical decision-making techniques and it is incumbent on the officers of the Navy to stay abreast of these developments. Therefore, the aim of this paper is to encourage the utilization of the school of thought that is emerging from the melding of the infant sciences of operations research, decision theory, and the older more established field of industrial management. Further, the aim is to show its applicability to aircraft squadron management. This discipline would have each manager take a new unbiased look at his objectives and plans and weigh each of his alternatives in a completely dispassionate manner. Usually some mathematical tools are encouraged since mathematics is a language that makes the transmission of biases most difficult and needs relatively few definitions.

In this paper the investigator will explore some of the more elementary techniques of industrial management and operations research where applications to a squadron can be beneficial. Applications, however, will be general and in conceptual form with a few illustrations. It is hoped that the reader's interest will be sufficiently aroused to study techniques further from authors whose technical competence is far superior.

This approach to problem solving is not new. It has been given numerous titles, probably the oldest of which is Scientific Management, bestowed by F. W. Taylor.[1] Persons in the field of Management Sciences will be quick to point out that Mr. Taylor's thesis was certainly far removed from the techniques and even concepts of today and that an enormous metamorphosis has taken place in management thinking since these early beginnings. Again, in this paper I shall not concern myself with the more esoteric nuances of the science of management. Rather the attempt here is to show applications to foster increased efficiency and effectiveness with little emphasis on the historical development of such beneficial concepts.

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[1]F. W. Taylor, The Principles of Scientific Management (New York: Harper and Brothers, 1915)

CHAPTER II

REVIEW OF THE LITERATURE

The proliferation of books, periodicals, and articles presently available on the practice of management is somewhat staggering. Not only is there a large quantity of material, but the quality varies considerably. A large portion of these materials are couched in mathematical terms that require more than average facility to comprehend. It is refreshing to note the increasing number of authors who, while utilizing the language of mathematics, write to the intelligent layman level. A note of warning is necessary, however, since it is somewhat the vogue for authors to state in the prefaces of their works that no real knowledge of the mathematics is required and then proceed to disprove such a statement in the first chapter. H. A. Simon took note of this tendency in his Models of Man when he stated,[1]

> It is a common whimsy among mathematicians to preface their books with the statement that the reader needs no particular mathematical training as prerequisite to the work in question but only a certain amount of "mathematical maturity". Since my own formal training in mathematics stopped somewhere short of the calculus, I was more than once the victim of such statements until I learned how deceptive they were and subjected myself to some hard study of mathematical textbooks--in the proper sequence and not failing to work out the exercises. "Mathematical maturity", it would seem, is acquired largely by studying mathematics.

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[1]Herbert A. Simon, Models of Man (New York: John Wiley and Sons, 1957), p. ix

The works referred to in this paper will not be of the type described by Simon and the reader will be warned when the subject requires more than a knowledge of algebra and an inquiring mind.

Mathematical Programming by Reinfeld and Vogel is an extremely good text and reference work for the novice.[2] The chapters on the Distribution Method and the Modi Method are lucid and do not require any background in mathematics. Of particular note is Vogel's Approximation Method which can be of use to solve rapidly the sort of problems that face a squadron operations or maintenance officer. It requires a knowledge of the Modi Method to insure an optimum; however it greatly diminishes the calculation time in problems that can be handled by pencil and paper techniques.

The chapter on the Simplex Method is outstanding and could be of assistance to officers finding a need for such a technique. It is not envisioned that numerous allocation problems requiring a tool as powerful as the Simplex will be encountered. Large units, such as Anti-Submarine Warfare Squadrons or Transport Squadrons, might find occasional need for such a technique. Mathematical Programming's chief virtue is that in it's treatment

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[2]Nyles V. Reinfeld and William R. Vogel, Mathematical Programming (Englewood Cliffs, N. J.: Prentice Hall Inc., 1958)

of the Simplex, no knowledge of matrix algebra is necessary. A simple procedure is clearly delineated which will obtain an optimum solution to the problem. Another distinct advantage is that sections may be read in any order without loss of continuity.

Several case studies are presented in which, although they deal with civilian business, the similarity to certain naval operations is apparent.

Scientific Programming in Business and Industry by Vazonyi is in the same category as Reinfeld's and Vogel's text.[3] The explanations tend to be a bit more general and therefore may not be as readily understood by persons lacking background knowledge.

A particularly commendable work is Quantitative Analysis for Business Decisions.[4] In addition to being a good all-around text, it contains in one of its chapters an excellent discussion of one of the most difficult problems faced by persons using the quantifying technique, that of establishing the utility of whatever is to be gained or optimized. The discussion deals with establishing the utility of money to a manager and clearly shows the train of thought that can be used to determine the utility of flight time or more training time, for example. The book is well written with the operating manager in mind. The chapter devoted to

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[3]Andrew Vazonyi, Scientific Programming in Business and Industry (New York: John Wiley and Sons, Inc., 1958)

[4]Harold Bierman, Jr., et al., Quantitative Analysis for Business Decisions (Homewood, Ill.: Richard D. Irwin, Inc., 1961)

probabilities is concise and would be an excellent primer for the beginner. Some mathematicians might disagree with the subjective treatment of probabilities given by the authors. In their defense they state,[5]

> Some mathematicians hold that it is improper to use probability measures in the personalistic or subjective manner just described. They argue that only objective probabilities for repetitive events have any true meaning. However, after examining the position of such people as L. J. Savage and R. Schlaifer, we feel that the use of such subjective probabilities is justified for purposes of decision making.

Further, this subjective appraisal of probabilities is the very essence of the discipline of thought that this paper will attempt to encourage.

Schlaifer's Probability and Statistics for Business Decisions is an excellent text and reference work.[6] Problems are solved in steps that are easily followed and the author explains the underlying rationale as well. This work is very useful as a reference since it contains distribution tables.

Analysis for Production Management by Bowman and Fetter has a well written chapter on the Monte Carlo Analysis.[7] This text may require application on the part of the reader to

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[5] Ibid., p. 7

[6] Robert Schlaifer, Probability and Statistics for Business Decisions (New York: McGraw-Hill Book Company, Inc., 1959)

[7] Edward H. Bowman and Robert B. Fetter, Analysis for Production Management (Homewood, Illinois: Richard D. Irwin, Inc., 1961)

comprehend without instruction; however it has the advantage of presenting in one book most of the more useful mathematical tools. The portions on statistical control and linear programming are more than adequate for the needs of squadron officers and should prove extremely useful to them. The Monte Carlo Analysis could be used extensively in squadrons since accurate records are kept in the unit; therefore construction of distributions from historical data is easily accomplished.

There are a large number of works available on linear programming. Some are quite extensive and have within them rigid proofs of the theorems. Most squadron managers will find, however, that the linear programming techniques in the works described above sufficient to solve the vast majority of his problems. In addition to this list, Gass's Linear Programming is suggested for those who wish to pursue the study of this class of problems further, either for their own edification or of necessity to obtain a greater depth of knowledge to solve their problems.[8]

One of the finer works that introduce the discipline without extensive use of mathematics is Operations Research for

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[8]Saul I. Gass, Linear Programming (New York: McGraw-Hill Book Company, 1958)

Management.[9] These two volumes are a collection of articles by the leaders in the field and provide an excellent background. Portions may be read without loss of continuity. The case histories are noteworthy and clearly exemplify the uses of operations research techniques in the military since a number of them are military studies.

Numerous works are available on production control and scheduling. The family of problems discussed is generally of the plant production type. While these are useful to officers dealing with some phases of the Naval Establishment, their benefit to aircraft squadrons will be minimal.

One of the basic tools of the industrial manager, progress charting, has already received wide application in all naval units. The credit for introducing charting as a technique into industry is ascribed to Henry L. Gantt, who fully described their construction and uses. Wallace Clark's The Gantt Chart is an encyclopedic text on the subject since it draws together all the publications by Gantt into one volume.[10] Even though charting has been part of the accepted routine of managing a squadron for some time, the methods in this work could be quite beneficial.

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[9]Joseph McCloskey and Florence Trefethen (eds), Operations Research for Management (Baltimore: The Johns Hopkins Press, 1954)

[10]Wallace Clark, The Gantt Chart (London: Sir Issac Pittman and Sons, Ltd., 1952-55)

A publication of great significance is Ewing's Long Range Planning for Management.[11] The section on nature and principles of planning and the section on the steps in making a plan are particularly applicable to the problem at hand. The problem of long range planning will be discussed at length in the succeeding chapter and the author will draw heavily on these sections. A method of using mathematical techniques is well exemplified in Chapter 20, an article by E. Leonard Arnoff on operations research in long range planning. This chapter might be more meaningful if the reader would read Chapter 7 of Bierman et al. first.[12]

The periodicals presently available that are orientated toward a mathematical approach to management in general demand a fair amount of background knowledge in order to grasp the subject under discussion. The usefulness is therefore dependent on the ability of the reader to comprehend and apply the solutions to his particular problem. Operations Research, published by the Operations Research Society of America, is an example of this. Contributing authors often submit studies of military problems that may be of use and interest to the squadron manager. Management Science, published by the Institute of Management

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[11]David W. Ewing, (ed.) Long Range Planning for Management (New York: Harper and Brothers, 1958)

[12]Op. cit. Chapter 7

Science, is oriented toward the civilian industry application and may not be readily applicable to use by an aircraft unit. The Harvard Business Review, although primarily for non-military managers, contains articles that are general enough in nature to be of interest. Gaumnitz and Brownlee have an article in that publication that parallels the aim of this paper among managers in industry.[13] They take the position that managers must learn some of the mathematical aids to be able to communicate with specialists in the field. Furthermore, an ability to formulate a problem will clarify the issue since attention will be directed to the constraints. In other words, merely identifying a problem as one of maximization or minimization and finding the limiting factors will greatly simplify decision making even if the problem cannot be solved mathematically.

While not really part of the literature, it seems appropriate to apprise squadron officers of the availability of trained operation analysts. These men are presently attached to each of the large fleet staffs. They are employed to analyze problems as they are assigned and are allowed to engage in what might be termed pure research, i.e. problems that they feel will lead to

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[13] R. K. Gaumnitz and O. H. Brownlee, "Math for Decision Makers", The Harvard Business Review, XXX (May, June, 1956) pp.48-56

increased efficiency or effectiveness. Their services, therefore, are not available on demand. However, if a squadron has a particularly knotty problem that is common to several units, aid may be obtained.

The literature cited in this chapter is not to be considered as extensive or comprehensive. It is an attempt to guide the footsteps of a beginner to some of the basic texts. Once an interest in scientific management and mathematical programming is kindled, the specific path to be followed is best left to the choice of the individual.

CHAPTER III

APPLICATIONS OF MATHEMATICAL TECHNIQUES TO AN AIRCRAFT SQUADRON

A method by which increased effectiveness and efficiency can be achieved in a unit is by planning. Planning is variously defined. Webster defines it as a scheme for making, doing, or arranging; a project; program. Billy E. Goetz gave a much more directly applicable definition when he said planning was "fundamentally choosing".[1] By the development of a long range plan it shall be demonstrated how the techniques that are encouraged can aid in this fundamental choosing.

A discerning observer inspecting an aircraft squadron will be struck with the lack of an overall plan. Plans rarely are made in any length, and when they are formulated, the scope is generally quite small. Usually all the major influencing factors are not considered, nor are a sufficient number of alternatives. Little integration of operating plans with administrative requirements or maintenance crew training is done, except where the requirements are so overpowering as to place them high on the priority list. Good examples are administrative and material inspections and service-wide examinations.

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[1]Billy E. Goetz, Management Planning and Control (New York: McGraw-Hill Book Company, Inc., 1949), p. 2

It is apparent that there is a need for examining all the requirements and alternatives and determining a method of spending the time and resources available in an optimum manner. By presenting the alternatives in this manner, the commanding officer can make a much more sensible decision. Generally commanders are well informed and have sufficient experience to recognize their command's deficiencies. However, they are not aware of the alternative ways of achieving their goals and have but a sketchy idea of what they must sacrifice in one area to increase the output in another. Lacking this knowledge, they cannot plan in a meaningful manner. In fact, some authors feel that without the use of alternatives there can be no planning at all. Koontz and O'Donnell state that if there are no alternatives in objectives, policy, or procedure, planning would be so rigid as to be nonexistent.[2] They point out that this is rarely the case in any organization.

Koontz and O'Donnell also offer a six step procedure for developing a long range plan. These steps are: (1) establish the objective, (2) establish the planning premises, (3) search for and examine the alternatives, (4) evaluate the courses of action, (5) select the course of action, (6) make the necessary derivative

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[2]Harold Koontz and Cyril O'Donnell, Principles of Management (New York: McGraw-Hill Book Company, Inc., 1959), p. 453

plans. This procedure gives an excellent framework on which to apply the newer techniques. Let us examine each step as it applies to an aircraft squadron and show how efficiency and effectiveness can be maximized.

I. ESTABLISH THE OBJECTIVES

Objectives are more readily thought of in a squadron in relationship to the mission. Each type squadron is required to train pilots to be as proficient as possible under the constraints of time and money. Further, each squadron is required to keep its aircraft in good condition and train its maintenance and flight crews, where they are utilized. Enlisted training can no longer be considered merely desirable since aircraft are increasingly complex. Failure to aggressively train may result in a unit having no one capable of performing the maintenance on the more technical equipment. The training under discussion is the technical training necessary for proper maintenance and repair of the aircraft and related equipment and is not to be confused with training of a more general nature required for satisfaction of the many directives of higher authority.

The establishment of a healthy work atmosphere is definitely part of the objectives of the unit. This is in consonance with the stated primary duty of the commanding officer, morale. The unit must be as responsive as possible to the human needs of its

members and under this heading another type of training is required.

Stated simply, the objectives of the squadron become, then, to be as efficient and as effective as possible in the performance of its mission and to cultivate a happy work atmosphere. Basic though this may be, it is important that these objectives be clearly stated and their meaning understood by all the members of the organization. Fleet Commanders have long emphasized the importance of each man knowing the mission of his unit. This emphasis is well directed and perhaps should be expanded to include entire objectives of the squadron. Some civilian concerns believe so strongly in this precept that they print and publish their objectives in the form of a creed. Perhaps an effort such as this would serve to keep all the members of a unit working toward the same goals and reduce sub-optimization. Sub-optimization is that phenomena which occurs when one group or section of an organization optimizes its own procedures to the detriment of the whole.

In summary, then, it can be seen that the first step in establishing a long range plan is to write out the objectives of the organization in some general and concise manner so that each member can see clearly the overall goal.

II. ESTABLISH THE PLANNING PREMISES AND SEARCH FOR THE ALTERNATIVES

To the units under discussion here, establishing the premises becomes much simpler if the goals are defined in a quantitative manner. Since a military unit is not free to decide for itself many of the things that drastically affect its operation, these goals must be carefully recorded. It is in this step that some new logic can be applied.

Consider the present state of affairs in which the pilot qualifications are set forth for the tactical phases by the type commander and the instrument flight qualifications are established by the Chief of Naval Operations. Further, there are requirements placed on the unit by the Task Force Commander in the form of standing and special operations orders. Aircraft availability, weather, funds, and the availability of a carrier are further constraints on the unit. There is a requirement for an administrative and material inspection as well as an operational one. The unit is expected to compete in fleet exercises and to provide some services to others in their exercises. It is clear that the most limited of all the resources is time. How, then, shall there be an ordering from this milieu? The premises and the search for alternatives will be considered simultaneously in this instance. The nature of the problems is such that many factors interact and influence the ultimate decision. By the concurrent study of

premises and alternatives, perhaps the decision making itself will be made simpler. This is where the discipline of thought shall apply. For the sake of illustration, a fictitious squadron and a limited number of these requirements will be used.

Assume a light jet attack squadron that has twenty pilots and twelve aircraft assigned. It has a crew of 140 men and has a six-month training period to be followed by a deployment. Its mission is such that only the following types of flight training will be considered: navigation, weapons, instrument, and field carrier landing practice. The officers of the unit establish that the minimum requirements in each of these categories is as shown in Table I.

TABLE I

TRAINING REQUIREMENTS

Type	Hours	Per Pilot
Navigation	100	
Briefing and preparation		50
Weapons	70	
Briefing and preparation		20
Instrument	35	
Briefing and preparation		20
Field carrier practice	20	
Briefing and preparation		10
All ground training	___	110
Totals	225	210
Total pilot training		435

There are six months in which to accomplish these minimums. Therefore there are 128 working days, since the commanding officer does not wish to work weekends unless it is absolutely necessary. The unit will make four moves which further reduce the time, one trip to an auxiliary base for weapon training, one carrier qualification trip, one fleet exercise, and one move aboard the aircraft carrier for deployment. Each trip will cause the loss of one day due to packing and unpacking. Transportation will be accomplished on weekends. The time lost due to these moves will be (1) four days lost for packing and unpacking, (2) ten days lost during carrier qualifications, (3) twenty days lost during the fleet exercises. The total days remaining for training is reduced to eighty-four days.

The commanding officer feels that each officer should spend forty per cent of his working time on his primary duty. Making this allowance, 84 x .60 = 50.4 pilot days available to perform the training, or 50.4 x 8 = 403.2 pilot hours. There is an obvious discrepancy of approximately thirty hours. This leads the commanding officer to make the following list of alternatives:

1. Delete thirty hours of pilot training.
2. Relax the constraint on the time allowed for officers to devote to their primary duties to 35.6 per cent and this will yield sufficient time to make up the deficit.
3. The unit can work overtime. The commander notes that if

on each day available for training the unit flies one four-plane sortie after the normal secure time for the unit, he could then gain $\frac{4 \times 2 \times 84}{20} = 33.5$ hours per pilot for the period.

This alternative looked very attractive when he reflected on it, since it would not require overtime on the part of the maintenance crew. Temporarily he selects alternative three as the optimum selection since he must sacrifice only eighteen hours of overtime per day.

The next constraint to inspect is that of aircraft availability. Past records show that the unit has had a very good average availability of approximately eight aircraft per day. The distribution is of interest however, and from the maintenance records Table II was derived from a one hundred day period.

TABLE II

AIRCRAFT AVAILABILITY

Aircraft Available	Number of Days
12	1
11	4
10	15
9	16
8	21
7	18
6	17
5	7
4	1
Total Days	100

From this it can be seen that a frequency distribution diagram (Table III) would show that forty-three per cent of the time there are less than eight aircraft available for flight and that only thirty-six per cent of the time are there more than eight.

The method of scheduling the use of aircraft is such that approximately 6.7 flight hours per plane day can be reasonably expected. Next construct a cumulative distribution diagram from this data and simulate one training cycle by the Monte Carlo method. While only one month will be demonstrated here, in actual practice several periods should be constructed. The method calls for establishing a cumulative frequency distribution diagram whose ordinate and abscissa is divided into one hundred equal units, plotting the relative frequencies cumulatively. Then by use of a random number generator we simulate the experience of the time period in question. Lacking a table of random numbers, the telephone directory of a metropolitan city may be used. In this instance, use the last three digits of the numbers selected from a random page. One should not attempt to write his own set of random numbers as it is extremely difficult to preclude a bias toward some given number.

Entering the diagram with the random number, the number of aircraft available on that day may be read off the abscissa. Continuing in this manner the experience for the time period is

simulated (see Table III). The length of time selected is twenty working days and the number of aircraft on any day as generated by the random number is displayed. From examining the information displayed here, note that only rarely does the average occur--in fact in the month simulated the average occurred only three times. The maximum was eleven and it occurred once. An important lesson to be learned from analysis of this sort is that in order to utilize the planes in such a manner to attain the average eight per day, the schedule must have provisions for the maximum available on any given day. From our random analysis, the commander decides to have a schedule that utilizes all the aircraft to which the maintenance officer assigns the smallest probability of being available for flight.

It is interesting to simulate a large number of periods and in essence allow the worst set of circumstances to occur. The whole argument for randomizing any distribution is to have on paper experience that would take years to document in actual practice. Thus by this simple technique we may allow the distribution to gang up on us, and in addition to seeing how often this occurs, we can see how best to counter these catastrophies.

Next determine the requirements for flight time. From Table I, page 19, the total is 225 hours per pilot, thus:

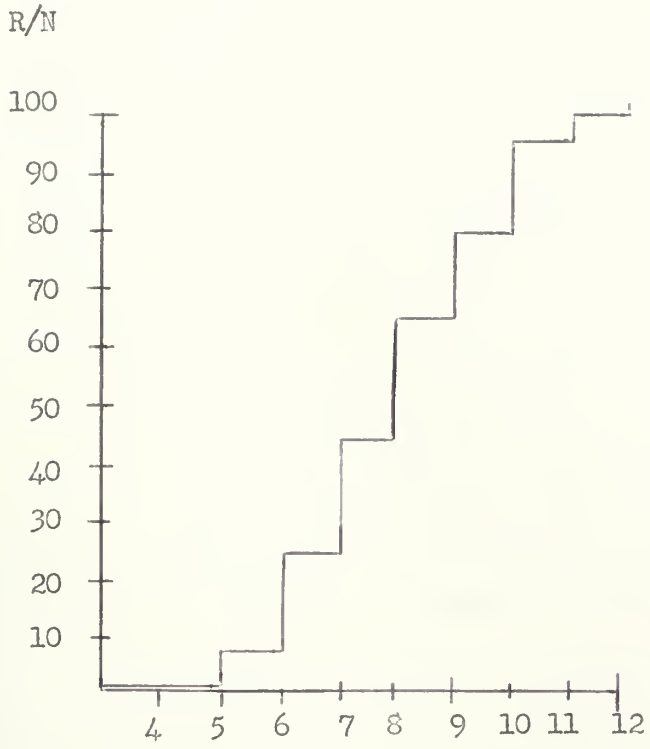
225 (hours) x 20 (pilots) = 4,500 pilot hours (3.1)

8 (aircraft) x 84 (days) = 672 aircraft days (3.2)

Dividing 3.1 by 3.2 we get 6.7 hours per aircraft per day.

TABLE III.

CUMULATIVE FREQUENCY VERSUS NUMBER OF AIRCRAFT AVAILABLE



Or construct a tableau:

0-1	2-8	9-25	26-43	44-64	65-80	81-95	95-99	99-100	Random No.
4	5	6	7	8	9	10	11	12	No. A/C

Random No.	A/C	Random No.	A/C
16	6	58	8
70	9	63	8
89	10	35	7
90	10	03	5
11	6	34	7
17	9	92	11
51	8	75	9
13	6	83	10
14	6	36	7
13	6	18	6

An inspection of the scheduling techniques in use in the unit revealed that such utilization was in fact expected with the extra sortie per day. Thus there is no need for further examination.

The largest variable in the problem of flight scheduling is that of the weather. Few aviation units, however, make use of the statistical weather information that is available to them. Each Naval Air Station maintains historical weather data that can easily be put to use. There is a need for only a statistical analysis of the past weather by broad types, i.e., was the weather below instrument minimums, instrument conditions, or did visual conditions exist? The data could be compiled by months and for enough years to insure that a fair sample had been taken. From these relative frequencies it can be deduced that the probability of a given type of weather will prevail a certain portion of the time. A large amount of literature has been written on the determining of the proper sample size in order to insure that the data is not biased. However in the case at hand, it will suffice to say that if a twenty-year sample is available for the months in question, then it should be used. Schlaifer has this to say about the establishment of probabilities from historical data.[3]

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[3]Robert Schlaifer, Probability and Statistics for Business Decisions (New York: McGraw-Hill Book Company, Inc., 1959)

In general, we shall assume it to be characteristic of rational behavior that:

If a person assessing the probability of a given event under a given set of conditions feels absolutely sure that the event would occur with relative frequency P in a very great number of trials made under these same conditions, he will assign probability P to the event.

While a very great number of trials is not possible in this case, a sufficient number should be obtainable. From the data Table IV is derived.

TABLE IV

PROBABILITY DISTRIBUTION OF WEATHER

Month	Below Minimum P(B.M.)	Instrument Conditions P(I)	Visual P(V)
1	0.05	0.2	.75
2	0.05	0.25	0.7
3	0.05	0.3	0.65
4	0.1	0.25	0.65
5	0.1	0.3	0.6
6	0.1	0.45	0.45

Since it is known when the unit will be making the trips for carrier qualification, and the fleet exercise, a table of expected hours that can be flown in the time remaining can be constructed. The expected flight time can then be compared to the requirements and the commander will then have a reasonably accurate evaluation of the overall plan. Inasmuch as the scheduling of the carrier qualification period and the fleet exercise are not controllable by the unit, they are assigned by higher authority to months three and five respectively. Therefore

Table V displays the combined scheduled time and the expected time to be attained. Entries in the hours scheduled column are adjusted for the loss of one day per move.

TABLE V

EXPECTED FLIGHT HOURS

Month	Hours Sched.	P(B.M.)	P(I)	P(V)	Hours I	Hours V
1	1,072	0.05	0.2	0.75	214	805
2	1,072	0.05	0.25	0.75	268	750
3	482	0.05	0.3	0.65	144	312
4	1,072	0.1	0.25	0.65	268	700
5	0	0.1	0.3	0.6	0	0
6	1,018	0.1	0.45	0.45	458	458
				Totals	1,352	3,005
Total instrument time plus visual time available						4,377

The trip to the weapon training base is not included in this table since it is the only factor over which the unit has some control. The weather at the weapons training base is such that it can reasonably be expected to be visual conditions during any month. Note that under the constraints that have been imposed, the unit is incapable of fulfilling its requirements even if it were able to utilize all the instrument time that is available for training. The problem is, then, to improve on the expected value by judiciously selecting the time for the trip to the weapon base where the probability of good weather is greater.

Consider the commander's own restraints as he contemplates the table; he does not wish to make the trip in the last month

before the deployment since he feels it would not accomplish the weapon qualifications early enough in the training period, months three and five are already involved with trips, leaving one, two, and four under consideration. By merely inspecting the table, it is obvious that month four is the best consideration for maximization of flight time. If this alternative is selected, the new flight time for month four will be as follows:

266 (5 days at the home base) - 53 (1 day packing time) = 213 hours

213 x P(I) = 53

213 x P(V) = 138

Total 191 hours available at home base plus

902 (15 days operation at the weapon base) yields 1,093 hours, an increase of 125 hours, and the requirements are feasible.

It has been established then, that there are enough hours of flight time available. The next consideration is whether or not the training can be accomplished in the type weather that will prevail. It appears that instrument conditions will prevail at the home base for approximately 1,350 hours of the scheduled time during the period. This factor is of prime importance.

In order to accomplish the training, the unit must fly all the hours possible for them to fly during instrument flight conditions. While this is a reasonable assumption to make in the early stages of establishing the premises, it can hardly be considered a logical assumption in view of the facts. Flight during

instrument conditions requires that each flight obtain a clearance with the controlling activities. Even more constraining is the increased space required between flights. This serves to reduce the number of aircraft that may be in the air at any given time. Further, takeoffs and landings are controlled; therefore a schedule made by the unit for maximum utilization of aircraft could not be met. Instrument flight conditions are established by limits of heighth of the base of the cloud layer and visibility at the base. There are, therefore, some days when this condition exists only locally over the base. During such conditions, when the top of the cloud layer is relatively low, instrument conditions presents no real problem to operation, provided of course, that the condition at the base remains above instrument minimums. When such is the case, climbouts and instrument approaches to the field can be accomplished fairly rapidly and the unit may conduct training of a type that requires visual flight conditions. On some portion of the days, the weather is such that only instrument flights may be conducted. The commander would like to have information on the relative frequency of the two types of instrument days, but he finds that while such information can be obtained, it is difficult to do so. Any reasonably thorough search of the records would require more time and effort than is available.

The meterologist, after some study of the records, states that a fifty per cent distribution would be within ten per cent

of being correct. Lacking better information, the commander elects to use the estimate and concludes:

1. Hours of flight time available for all purposes 673
2. Hours available for instruments only 673

Of the 673 hours of instruments only, 400 can be utilized since 300 hours, or 15 hours per pilot must be flown under simulated conditions. Thus there is an allowance for loss of maximum utilization of approximately forty per cent built into the case. There is no need to consider any alternatives. If the 400 hours of instrument training are flown when conditions dictate, the best utilization of time will be accomplished.

The requirements and constraints developed thus far by properly selecting the trip to the training base and by allowing for the loss of flight time due to instrument weather are as follows:

Total time available	4,502
Less allowance for weather	273
New total	4,229
Total required	4,500
Deficit	271

The alternatives developed to meet the requirements of total pilot training may be considered in conjunction with the shortage of available flight time. With the decision to fly one sortie after working hours, the time available is still 271 hours

too small. The alternatives are as follows:

1. Delete pilot training the three hours per pilot in the first instance plus the additional 271 hours, or 13 hours per pilot for a total of 43 hours per pilot.
2. Delete only the 271 hours or 13 per pilot by electing to fly the additional flight per day.
3. Work more overtime either on a daily basis or on weekends.

If consideration is given to flying on Saturdays, the probability of achieving the goal is of interest. Two conditions must be met in order to fulfill the requirements. There must be at least eight aircraft available for flight and visual weather conditions. The joint probabilities of these events occurring is the product of the independent probabilities. Thus

$$P(8a/c \text{ and } V) = P(8a/c) \times P(V).$$

The expected value or the number of Saturdays on which these two conditions can reasonably be expected to occur is given by the product of the joint probabilities and the number of Saturdays available. $P(8 \text{ and } V) \times \text{Number of Saturdays} = E(S)$. From Table II, page 21, and Table IV, page 26, Table VI can be constructed.

If the commander wished to obtain 256 more flight hours, the unit could do so by working five Saturdays during the period at 53.6 hours per Saturday and satisfy the requirements. From Table VI it can be seen that in months three and four the possibility of having the proper conditions is somewhat less than in

the other three. However this is a matter to be decided by the utility attached to the satisfaction of requirements.

TABLE VI

NUMBER OF SATURDAYS EXPECTED THAT FULFILL THE REQUIREMENTS

Month	P(S)	P(V)	P(S and V)	Number of Sats.	E(S)
1	.57	.75	.42	4	1.6
2	.57	.70	.4	4	1.6
3	.57	.65	.37	2	.7
4	.57	.65	.37	2	.7
5		Deployed			
6	.57	.45	.25	4	1.0

By the use of little more than arithmetic, the premises and the alternatives are exposed for critical and objective appraisal.

III. EVALUATE THE COURSES OF ACTION

A technique that may be of assistance in the evaluation of alternatives is the assignment of utilities. Utility as discussed here will be in the sense that von Neumann and Morgenstern apply to the term.[4] The steps in finding the utility that one places on a commodity are easily followed. The one note of caution is that in any utility rating the scale is entirely arbitrary and subjective, therefore applicable only to the person

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[4]von Neumann and O. Morgenstern, The Theory of Games and Economic Behavior (Princeton University Press, 1944)

for whom it was constructed. The rationale here will be patterned after Bierman et al.[5]

First formulate an alternative that promises two different rewards with equal probability. For instance, if the payoff were in money, then one could use $0 and $10,000. To each of these payoffs assign a utility index U. The selection is entirely arbitrary with respect to the range of the two numbers. The only restriction is that the more positive amount have a larger number than the smaller one. Thus if the utility indexes were 0 and 1 respectively, then it can be seen that

$$E\ U(A) = \tfrac{1}{2}U(\$0) + \tfrac{1}{2}U(\$10{,}000) = \tfrac{1}{2}(0) + \tfrac{1}{2}(1) \text{ and } U(A) = \tfrac{1}{2}.$$

Next select an alternative that will occur with certainty and ask the decision maker to choose between the alternatives. Continue to pose choices until you find the amount that will occur with certainty that the decision maker finds exactly as attractive as the original alternative A. This amount, then, has a utility of exactly one half, and three points have been established on the utility function of the decision maker. By continued application of this logic as many points as needed can be established.

For illustration of the use of utilities, consider the information in Table VII. The problem under consideration is

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[5]Harold Bierman, Jr., et al., Quantitative Analysis for Business Decisions (Homewood Ill., Richard D. Irwin, Inc., 1961)

whether or not to attempt to fly Saturdays early in the period when there is a good probability of achieving the goal, or attempting to fly one Saturday each month. The utilities and the states of nature are as displayed in Table VII. Utilities and probabilities will be arbitrarily assigned in this illustration. The decision maker assigns a utility of 6 to scheduling every available Saturday commencing at the beginning of the period until five are attained, given that the weather will be favorable at least one Saturday per month. With the condition of poor weather in two months and scheduling early, he assigns a utility of 12. To the alternatives of scheduling one Saturday per month under the same circumstances, he assigns 10 and 2 respectively.

TABLE VII

UTILITIES VERSUS STATES OF NATURE

Act 1 Sched. Early	Act 2 Sched. one per month	P States of Nature
6	10	.7
12	2	.3

The expected utility of Act 1 is then .7 x 6 + .3 x 12 = 8.2 and the E(U) of Act 2 is .7 x 10 + .3 x 2 = 7.6. The decision maker then may maximize the expected utility by selecting Act 1. Often the probabilities of the various states of nature are equally likely and are best solved by the use of game theory. This method is fairly limited in application since in games of any

size the computation becomes quite tedious. The Compleat Strategyst has an absorbing and easily understood treatment of this subject in non-mathematical terms.[6] Games of the order of five acts and five states of nature may be solved by the Simplex Method without too much difficulty.

Merely assigning utilities to the various acts should prove to be an immense aid to decision makers in squadrons. It must be remembered that such an analysis is entirely subjective and that the commander must assign the utilities if he is to be the decision maker. In general, the use of a technique such as has been described here should prove valuable in the evaluation of alternatives. If the alternatives are few and the choices limited, then selection is not difficult. Schemes such as the one above should be reserved for the more difficult choices.

IV. SELECT THE COURSE OF ACTION

The selection of the course of action will in great part be forced by the solution of the steps taken thus far. This is the appropriate place to note that the most elegant mathematical and probabilistic models may not fully satisfy the needs of the decision maker. There are often very small personal biases that

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[6]J. D. Williams, The Compleat Strategyst (New York: McGraw-Hill Book Company, Inc., 1954)

may be considered at the time of selection. Again in the use of linear programs, often the solution contains more than one method of maximizing the objective function. This is the area of personal perogative.

V. MAKE THE NECESSARY DERIVATIVE PLANS

In the same manner as the master plan was evolved, the derivative plans must also be formulated. There are many problems in the typical squadron that lend themselves to these techniques. Some of the more obvious ones are the scheduling of maintenance and the allocation of specialists in the maintenance crew. These two problems are the source of many hours of concern. No general solution can be derived for use by all squadrons since the particular skills needed by each of them is different. The use of the mathematical approach will, however, provide the officers concerned with a new and improved understanding of their problems. The essence of this school of thought is perhaps best stated by Reinfeld and Vogel in the form of some basic questions.[7]

1. What is the essential nature of the operation?
2. What are the choices?
3. What data must be gathered?

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[7]Nyles Reinfeld and William Vogel, Mathematical Programming (Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1958) p. 151

4. What are the complications in the present operation? How many can be alleviated by careful analysis?
5. How critical is timing? Is there time available for a careful study of the problem or must the decision be made now?
6. Who knows most about the overall operation? Who has the data?
7. What are the objectives of the operation? What is the goal that is being sought?
8. How can mathematical programming really help the operation? Remember the objective is to attain the goals, not to apply the techniques.

The problem presented here has been but one section of the whole plan. In order for a commander of an aircraft squadron to realize the full benefits of a mathematical program or model of his unit's operation, he must follow a similar process for each of the goals that is sought. Quantitative expression of each of these goals is to be desired. If such an expression is derived, it may prove that the factors are interrelated. The relationship is most likely to occur in the form of a system of inequalities which lends itself to solution by the Simplex Method. On the other hand, if such a relationship cannot be established, then each section of the plan may be optimized and its effect on the other goals studied.

Utility indexes are useful in the comparison of alternatives where some probability is involved. The entire problem of planning is probabilistic in nature, therefore judicious application of the utilities and expected utilities can be extremely illuminating.

In summary, then, it is the reasoning process that can be of great and lasting benefit to the officers of a squadron and not necessarily intricate computational procedures, inasmuch as each unit will be forced to create a model and utility indexes for its individual use.

CHAPTER IV

SUMMARY AND CONCLUSIONS

There is a need for increasing the abilities of officers in squadrons to properly manage their units. The pressure of advances in technology and the requirement for a higher degree of readiness are making this need acute. As the technology of aerial warfare increases and brings with it the enormous complexities of modern weapons systems, more definitive and positive methods of management must be utilized.

Civilian managements, when faced with a complex problem, often hire the services of expert consultants. Squadron managers do not have the authority to hire outside aid, though they may solicit assistance from higher authority. The use of the services of consultants for the smaller commands may be found necessary in the future. The present-day problems, however, cannot be deferred. Self aid is the only aid forthcoming in the immediate future.

Mathematical techniques are being used today in businesses that have management problems more difficult than those encountered in aircraft squadrons. A great deal of literature and scientific endeavor is concerned with the development of techniques that will aid the manager in selecting a course of action. Most of the techniques developed for businesses employ dollars

as a measurement of effectiveness. Profit is maximized or cost is minimized. Some of the functions of an aviation unit may be related to cost, however most cannot be directly related to dollars. In the fictitious problem posed in Chapter III, the use of time as a measurement of effectiveness is illustrated. Time is the most vital resource at the disposal of the aviation unit commander. The wisdom with which this precious commodity is utilized will determine the efficiency and effectiveness of the unit.

It can be seen, then, that mathematical programming techniques will definitely assist in the management of an aircraft squadron. It is not envisioned that on some future day a model representing all the variables will be written and that managing a unit will become simply a matter of solving the indicated equation. However, within some limits, the use of mathematical models will certainly improve the efficiency and effectiveness of any squadron. The amount and methods used are a matter that is best decided by the particular case. The planning of a unit's use of time and resources cannot help but benefit by the application of probabilities. Likewise, the definition of objectives is certainly improved by the use of utilities. The use of these two concepts alone will bring an improvement to the establishment of plans and objectives within aircraft squadrons.

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